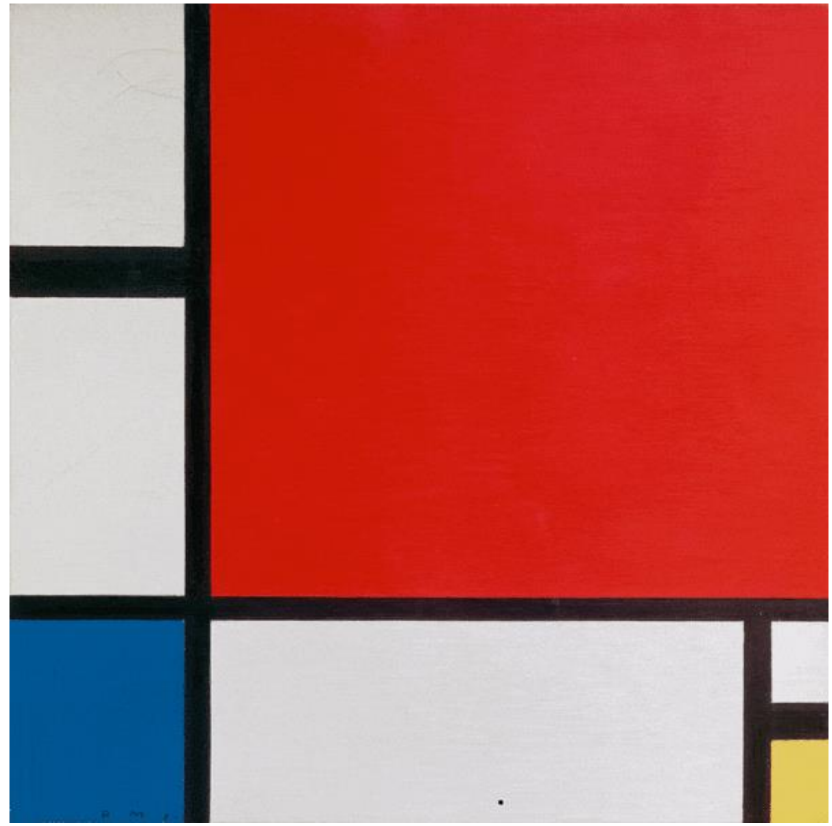
# Modernist Motion: Symbolic Dynamics as a Tool for 21st Century Science

**Arnaldo Rodriguez-Gonzalez** [†]



***Cornell Applied Dynamics Seminar***

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[†]*Field of Theoretical and Applied Mechanics, Sibley School of Mechanical & Aerospace Engineering, Cornell University*

Cornell University
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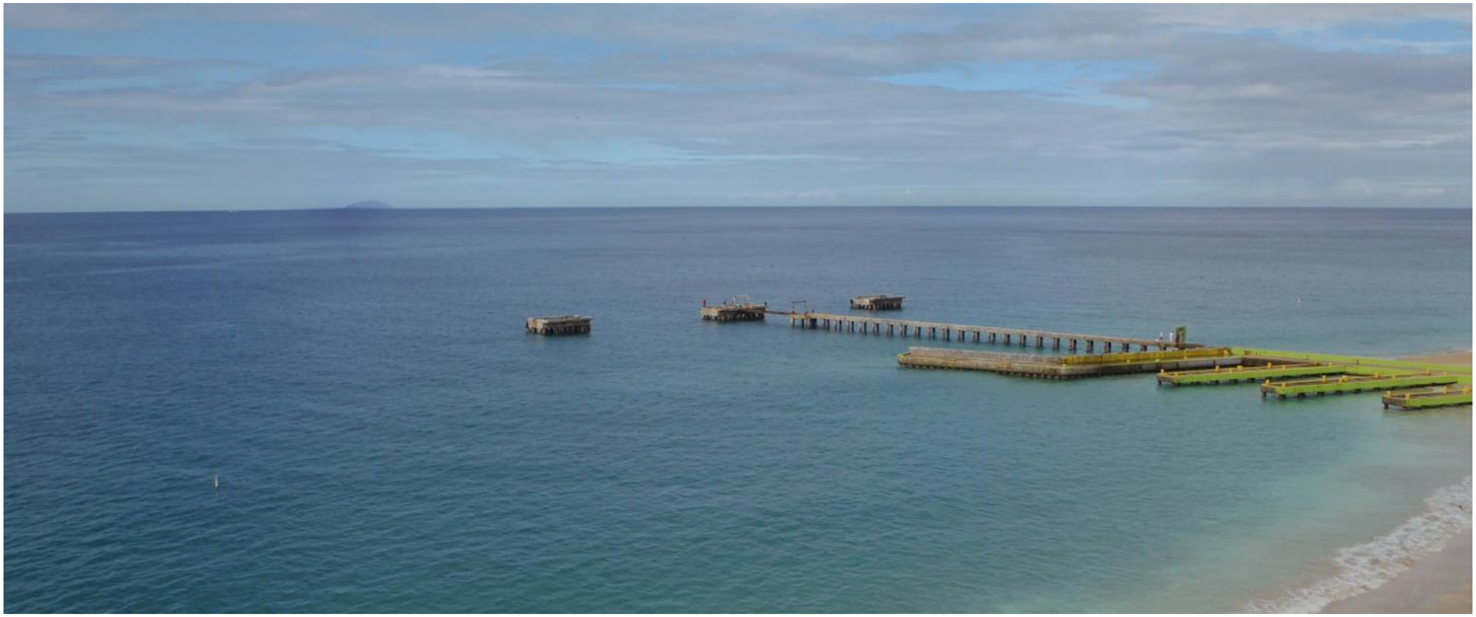
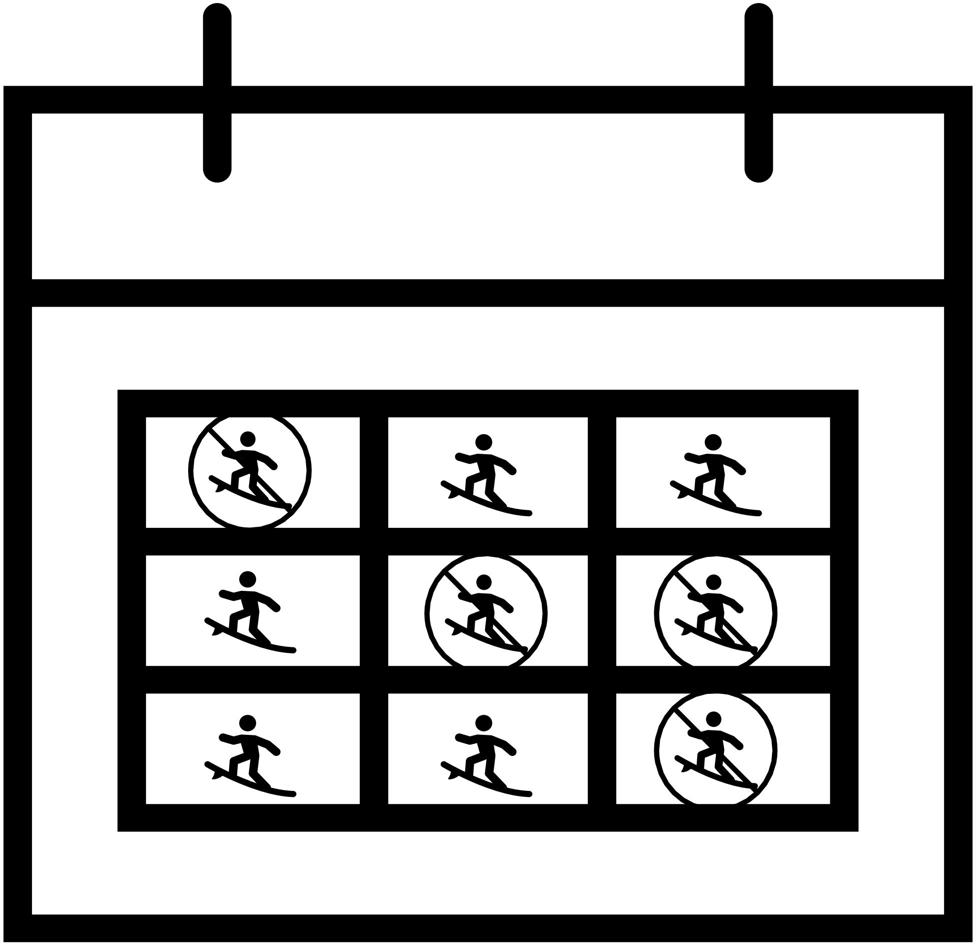
# Prelude: Surfing in the Caribbean

- **You want to go surfing off the coast of Puerto Rico sometime soon but don't know when it'll be a good day for that.**
  - **What a good day for surfing is depends on a great deal of environmental/social factors!**



# Prelude: Surfing in the Caribbean

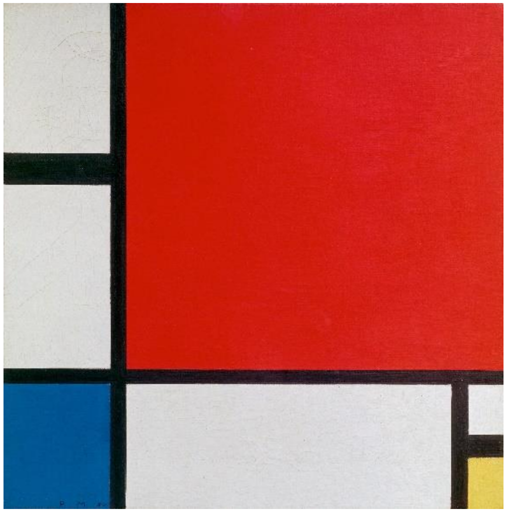
- **What could the surfing "calendar" look like?**



# Why "modernist" motion?

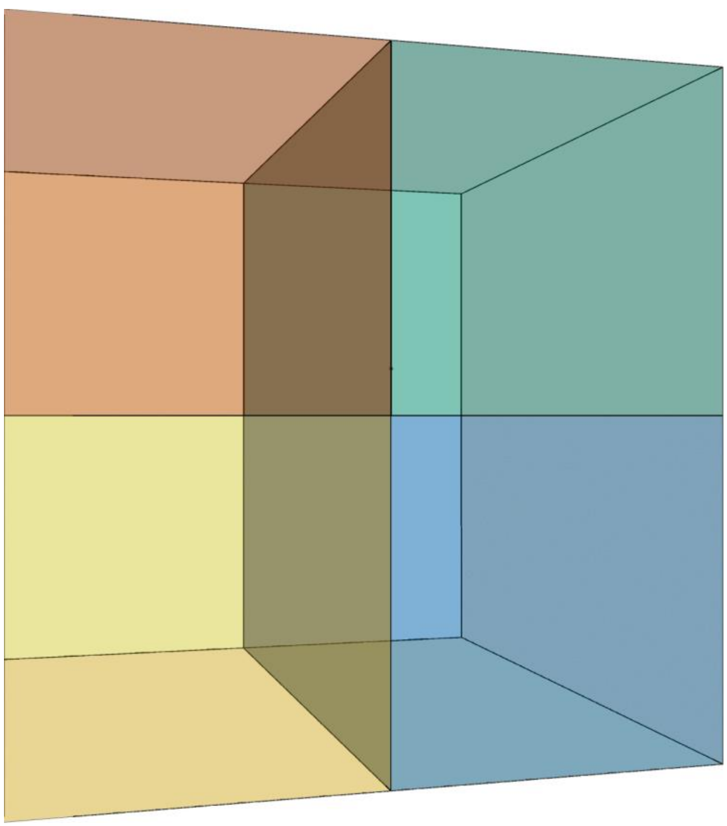
- **"I don't want pictures; I want to find things out." –Piet Mondrian**

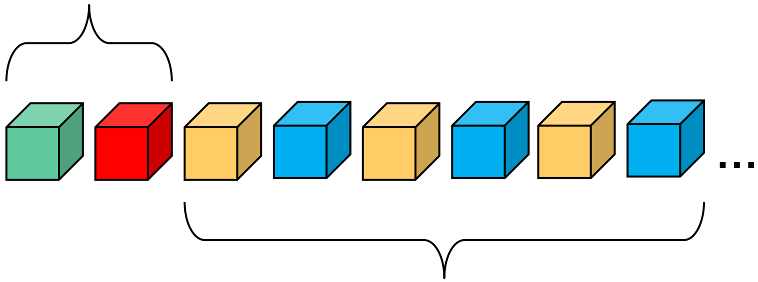
- **"The abstract painter considers the realist painter to be the abstract painter and himself the realist, because he deals realistically with the paint and does not try to transform it into something it is not." –Jimmy Leuders**



# Symbolic dynamics is a tool to extract meaningful information from complicated dynamical models

- **Symbolic dynamics is the study of sequences of symbols and their correspondence to complicated dynamical systems.**





# The use of symbolic dynamical tools follows a straightforward analytical process

1. **What symbols do we use, and what do they represent?**

2. **What sequences of symbols are allowed in our system?**

3. **What criteria need to be satisfied for a system to generate specific symbol sequences?**

# The “modernist” ideas of symbolic dynamics have been frutiful in the study of dynamical systems

- **How different can trajectories in the system be?**
  - **Topological entropy**
- **How does a dynamical map “naturally” partition the space it occurs on?**
  - **Symmetry breaking, Poincare-Bendixson theorem**
- **Symbolic dynamics is effectively the only way mathematicians can currently diagnose chaos.**
  - **Correspondence to shift map on space of arbitrary sequences.**

# Symbolic dynamics can illustrate key unintuitive features of chaos

**Chaos isn’t just a property of motion.**
**It’s a property of motion *in a region*.**

**To prove the motion of a car is chaotic, you need to know where it’s driving from/to.**

# Symbolic dynamics can illustrate key unintuitive features of chaos

- **We can describe the trajectory of the car as a sequence of visits around distinct regions (houses).**
  - **A possible trajectory: Green -> Red -> Blue -> Black -> Red -> Blue -> ….**

# Symbolic dynamics can illustrate key unintuitive features of chaos

- **For motion to be chaotic, each house needs to be arbitrarily close to another house.**
  - **Sensitivity to initial conditions**

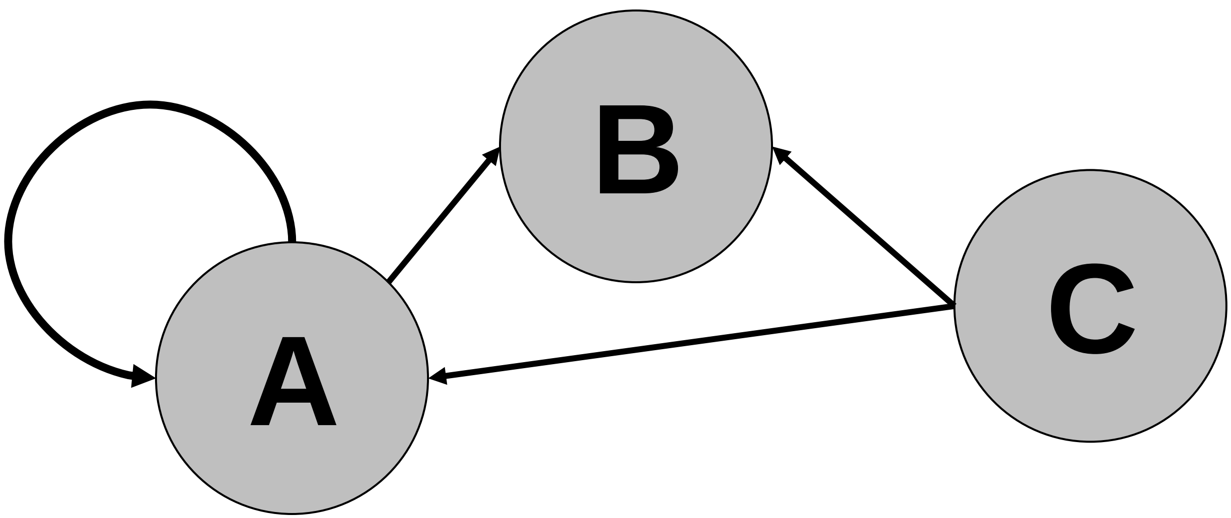
# Symbolic dynamical thinking demonstrates the connection between chaos and fractal structures

- **All these things together almost always causes the "neighborhood" to possess a fractal structure!**
  - **Individual regions = disconnected set**
  - **Arbitrary closeness of those regions = dense-in-itself set ≈ perfect set (if closed)**
  - **Only disconnected perfect compact set is the Cantor set up to a homeomorphism!**

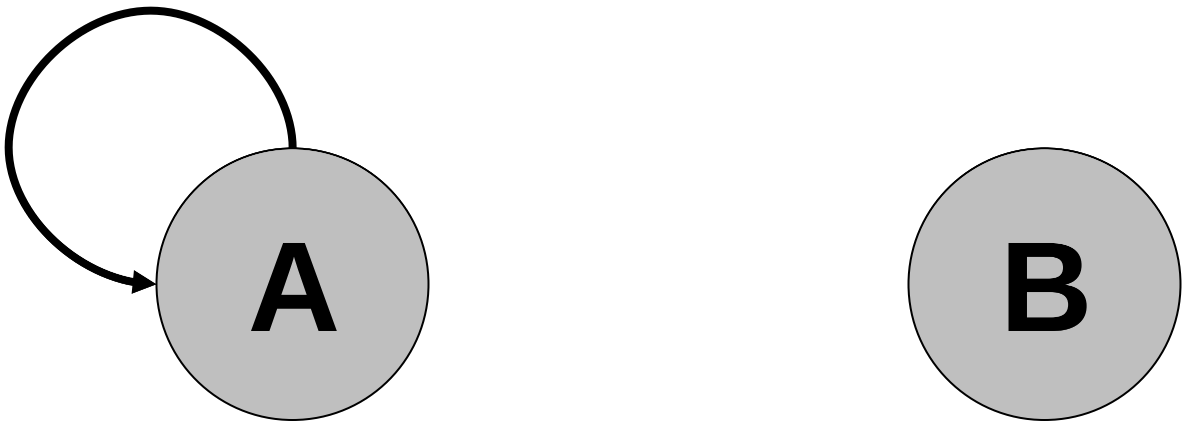
***Chaotic motion within a closed, bounded set happens inside of a fractal.***

# Symbolic dynamics can also be understood through graph theory

- **Directed graphs are an alternative to symbol sequences for describing dynamics in a symbolic way.**
  - **Vertices can represent regions or states**
  - **Directed edges can represent possible transitions**



# You can use symbolic dynamics to prove highly counter-intuitive dynamical behaviors




**Only period 1 orbits possible**

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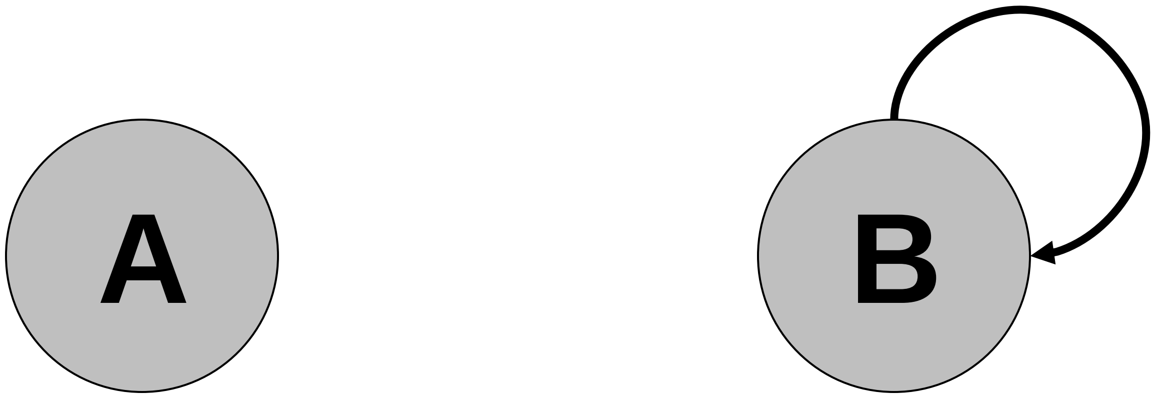


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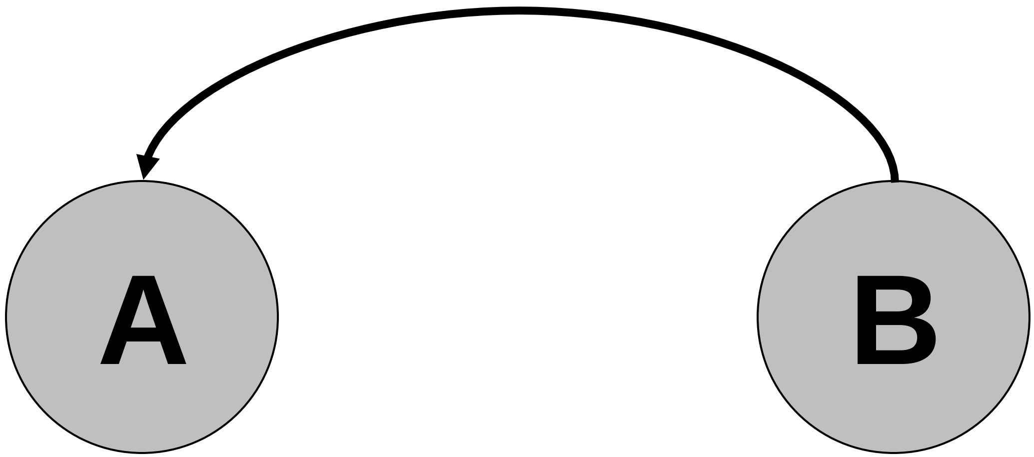


**Only finite orbits possible**

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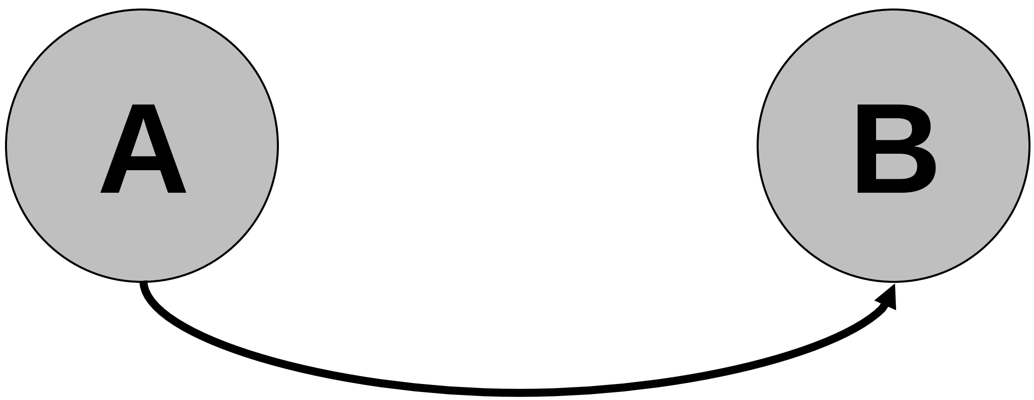


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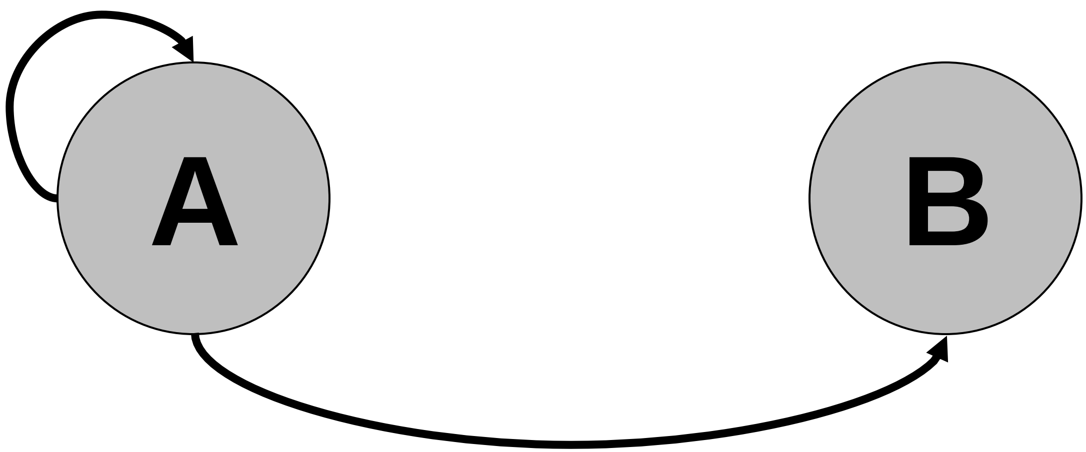


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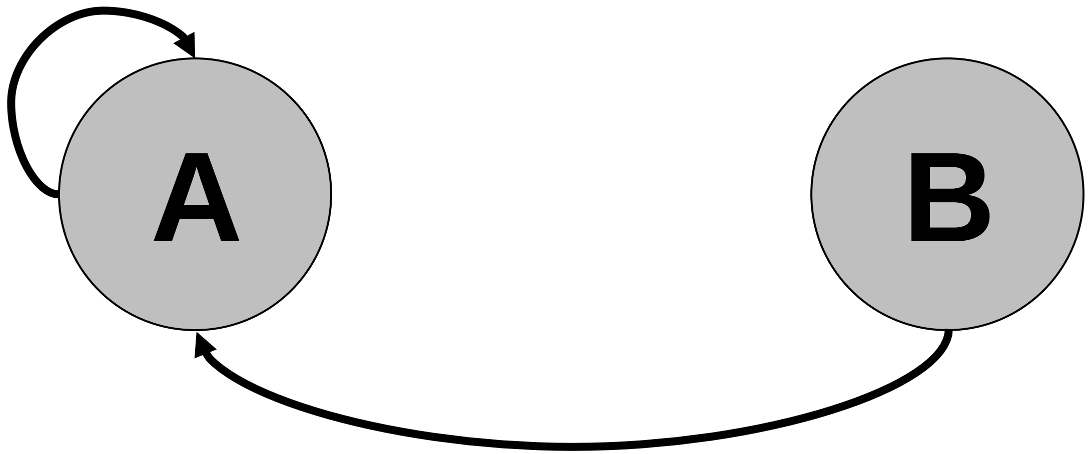


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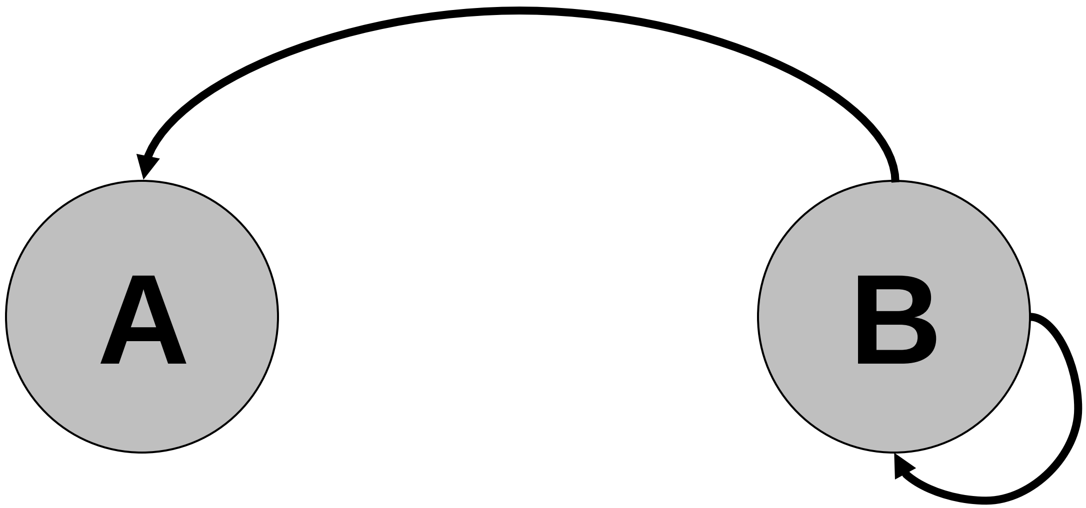


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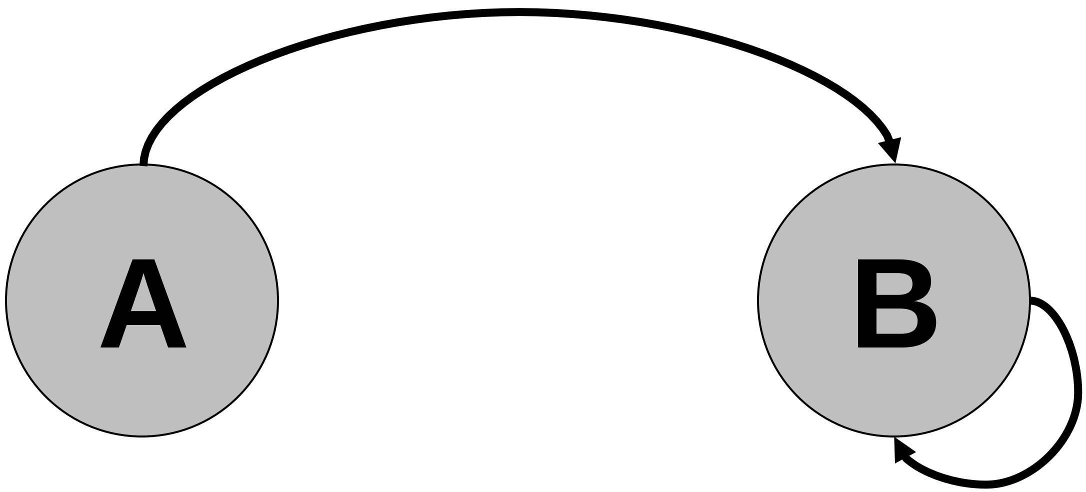


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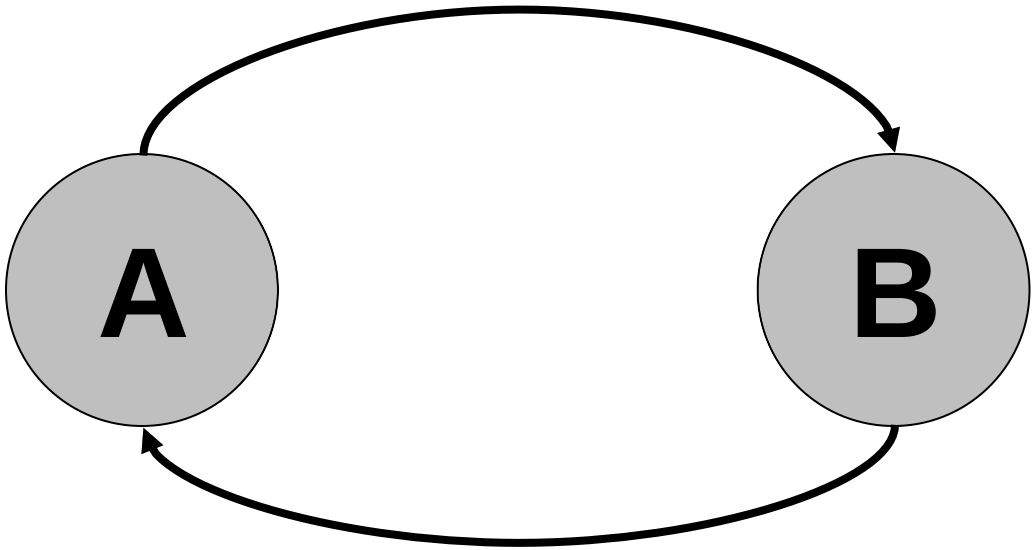
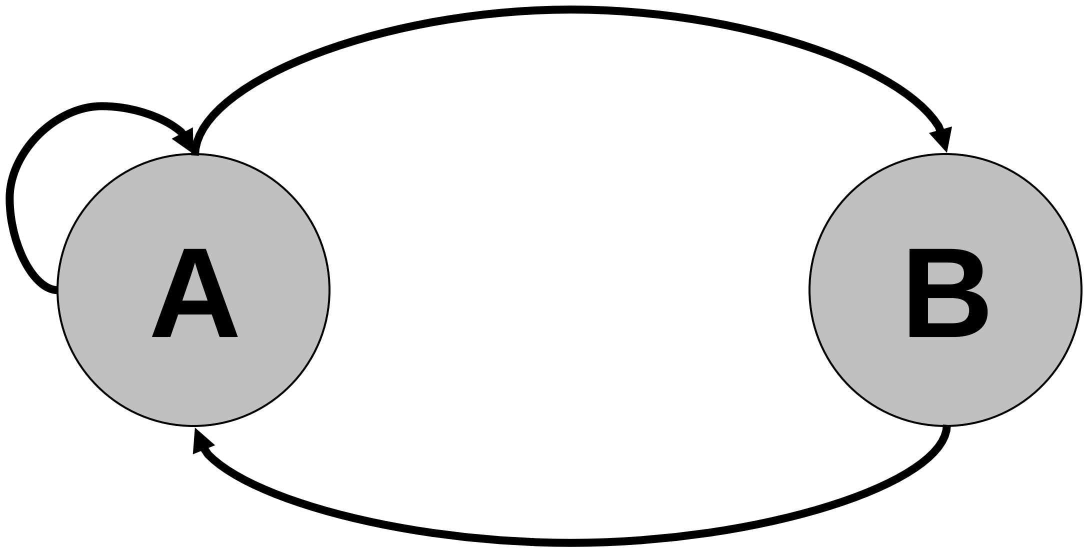



**Only period 2 orbits possible**

## You can use symbolic dynamics to prove highly counter-intuitive dynamical behaviors




**Period 3 orbits are possible; but so are orbits of any other period!**

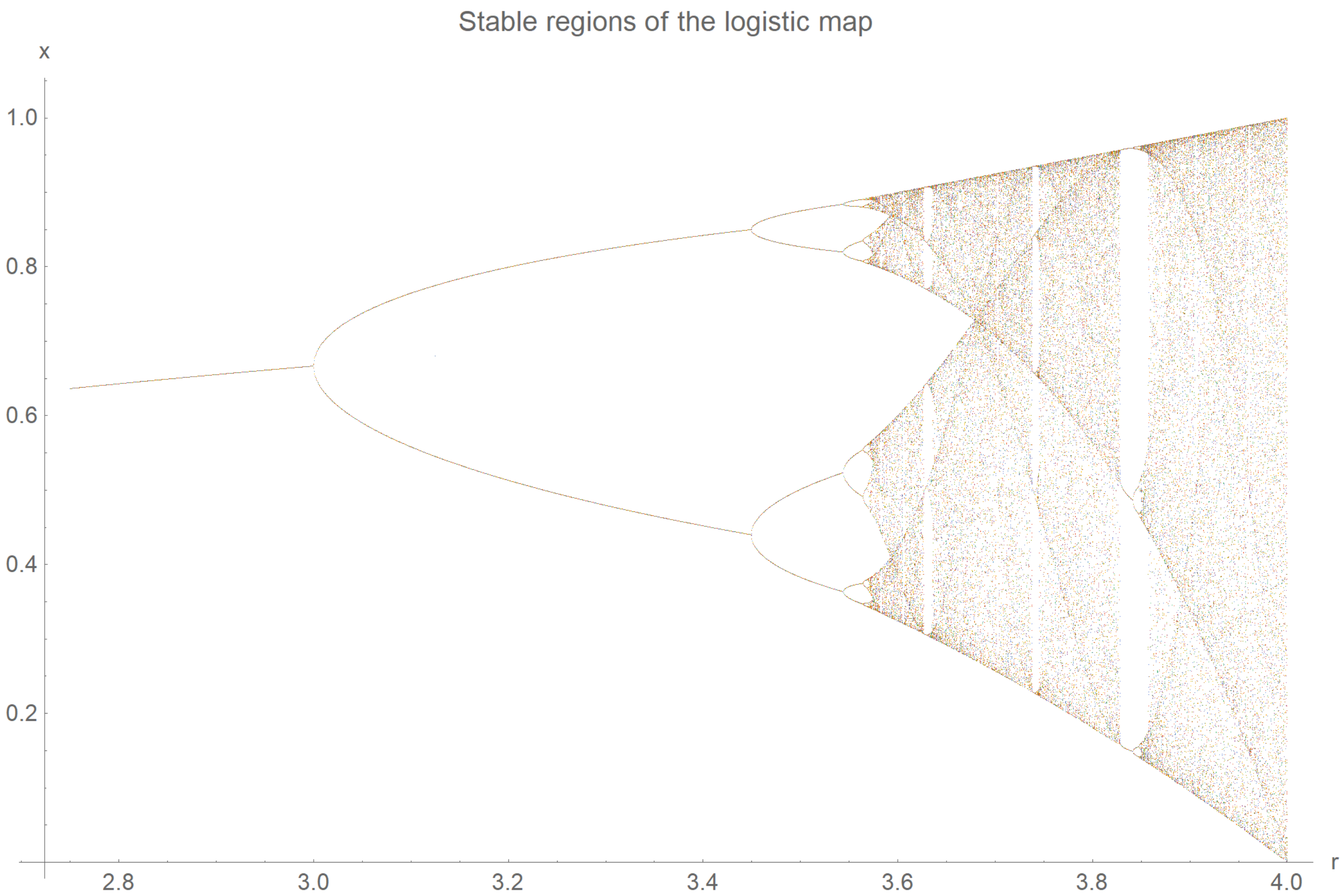
# You can use symbolic dynamics to prove highly counter-intuitive dynamical behaviors

- **Applications in stochastic processes**
  - **How likely is it that I visit a periodic sequence of states of some period?**

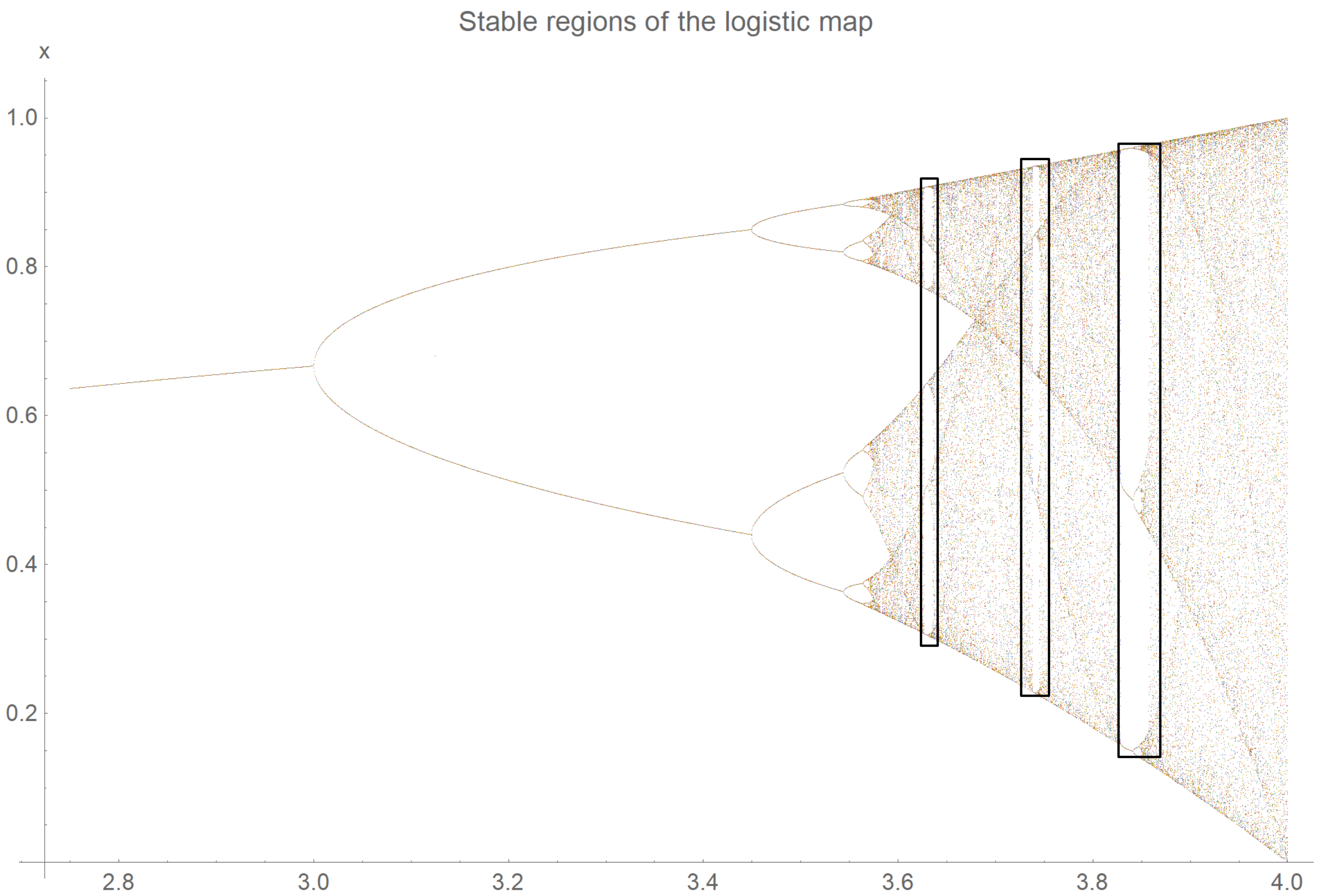
- **Li-Yorke or period three theorem: "Period three implies chaos."**
  - **Specific case of Sharkovsky's theorem**

- **Rodriguez-Gonzalez corollary: "Proofs of Sharkovsky's theorem implies nightmares."**

# The logistic map, $x_{n+1} = rx_n(1-x_n)$, possesses stable periodic windows after a transition to chaos



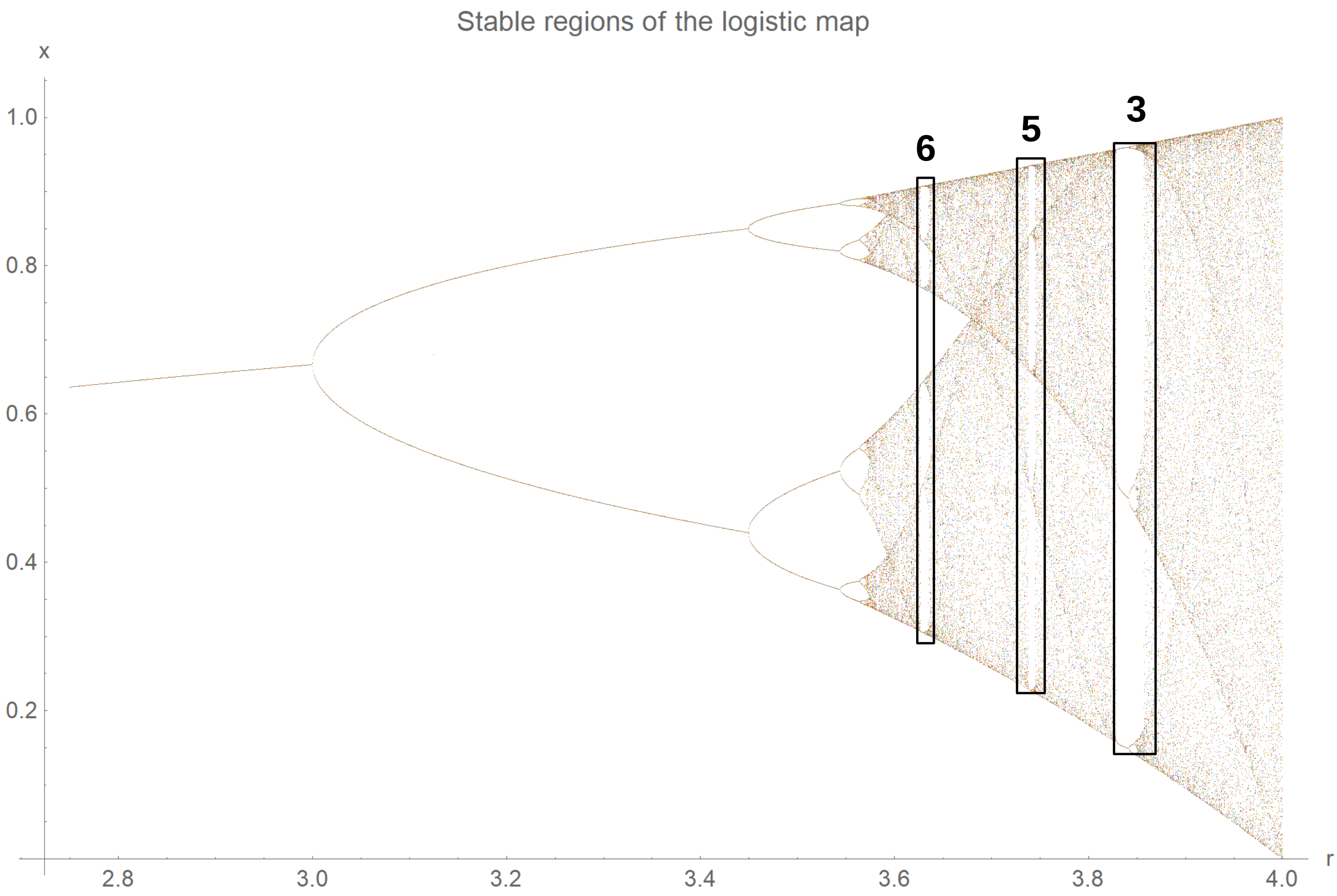
# The ordering of the periods of these windows does not appear straightforward



# Periodic windows for the logistic map can be determined using symbolic dynamics via a U-sequence

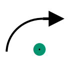
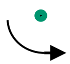
| Period (<7) of stable cycles | Symbol sequence |
|---|---|
| 2 | CR |
| 4 | CRLR |
| 6 | CRLRRR |
| 5 | CRLRR |
| 3 | CRL |
| 6 | CRLLRL |
| 5 | CRLLR |
| 6 | CRLLRR |
| 4 | CRLL |
| 6 | CRLLLR |
| 5 | CRLLL |
| 6 | CRLLLL |

# Periodic windows observed numerically in the logistic map follow the order predicted by the U-sequence

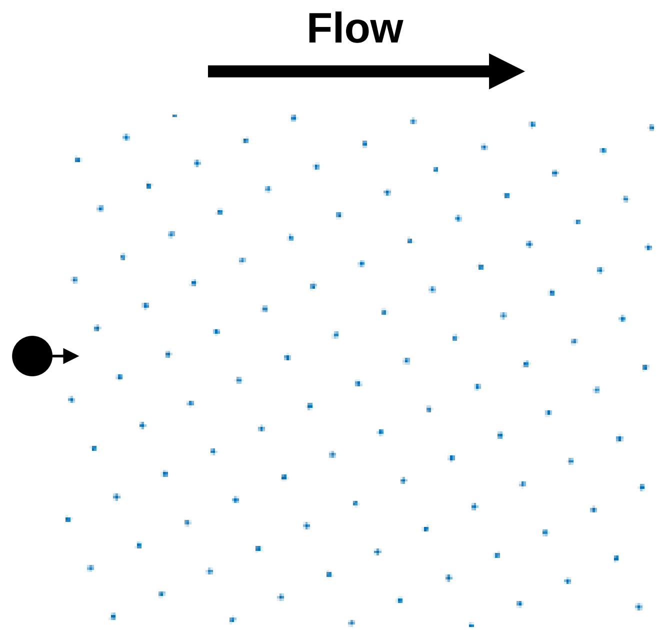


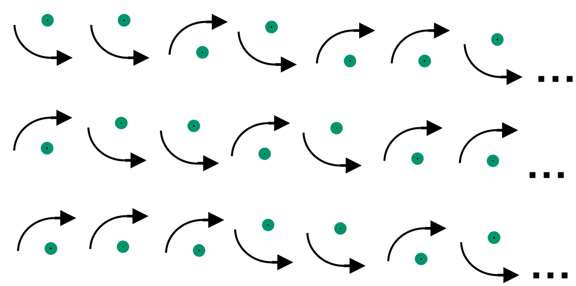
# In my research, I studied the dynamics of colloidal particles through microfluidic obstacle lattices

- **Let’s say you have a colloidal particle flowing through a microfluidic obstacle lattice.**

- **After colliding with an obstacle, it either “plinks” up or down.**
  -  or 

- **The no. of possible trajectories is almost infinite!**



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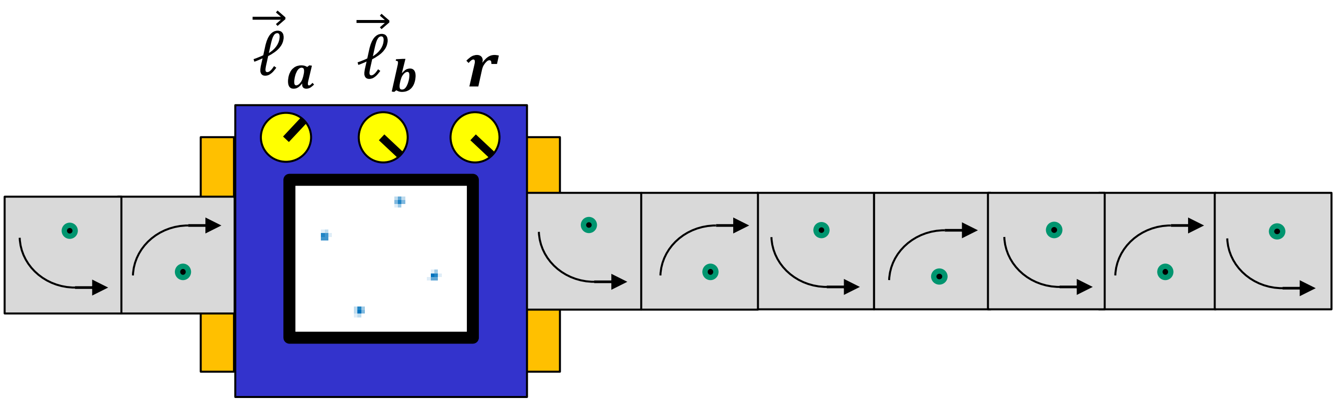
# Modeling colloidal particle advection through lattices as a discrete sequence of collision outcomes provides clarity on possible trajectories

- **A lattice functions like a computer, outputting a sequence of collision outcomes based on its parameters and the particle size.**

- **What properties does the lattice/computer have?**



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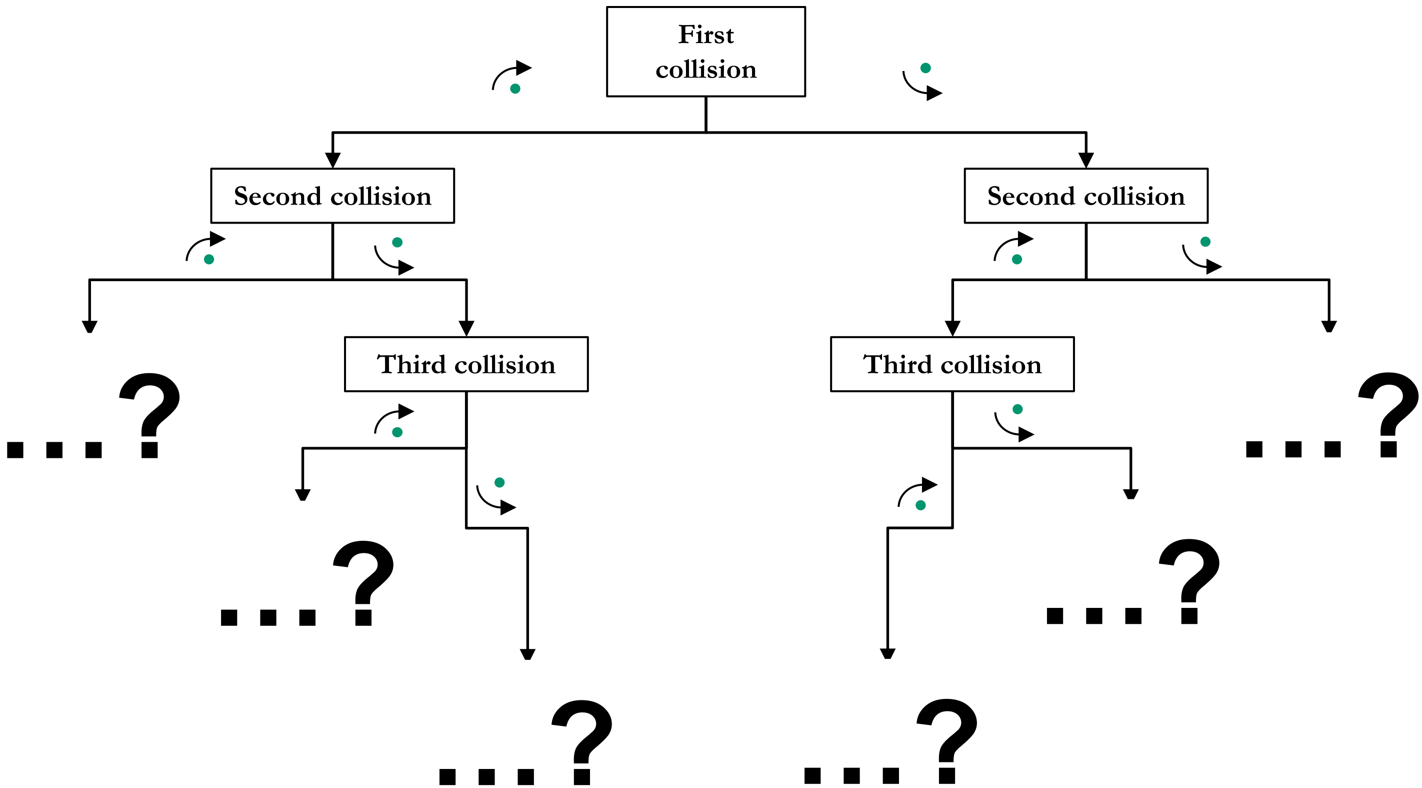
# The spatial symmetry of the collisions heavily restricts the kinds of paths the particles can take

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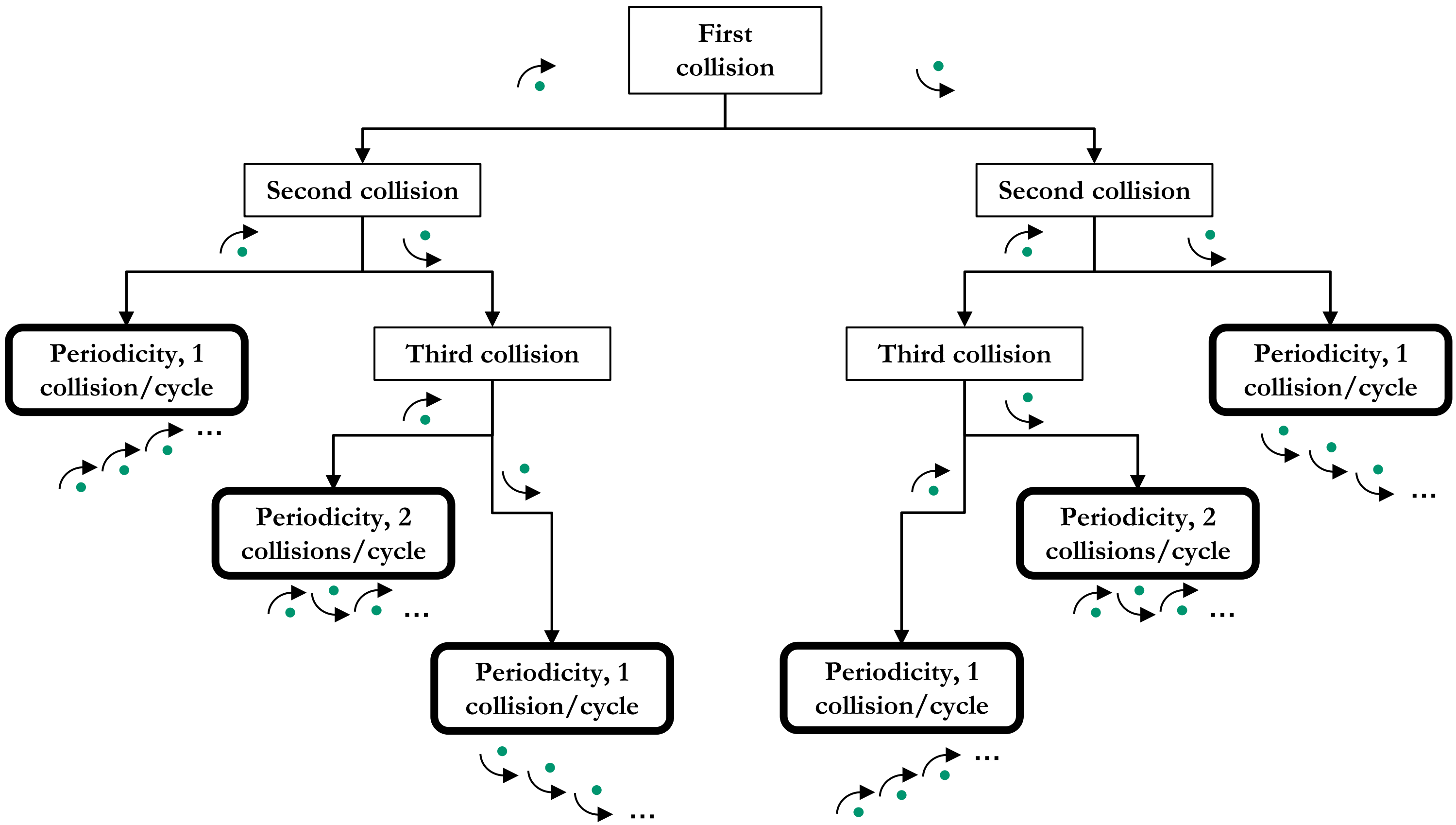


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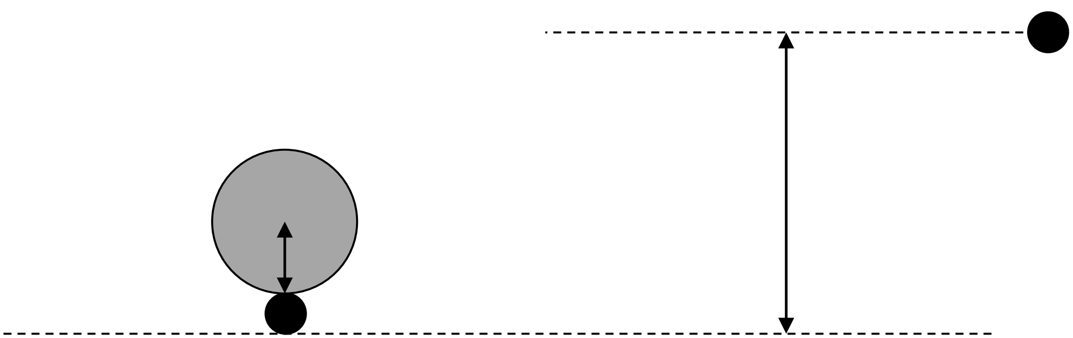
# After determining which orbits are possible, we can determine the conditions needed for each to occur

- **Under what conditions does a particle follow a period 1 orbit?**





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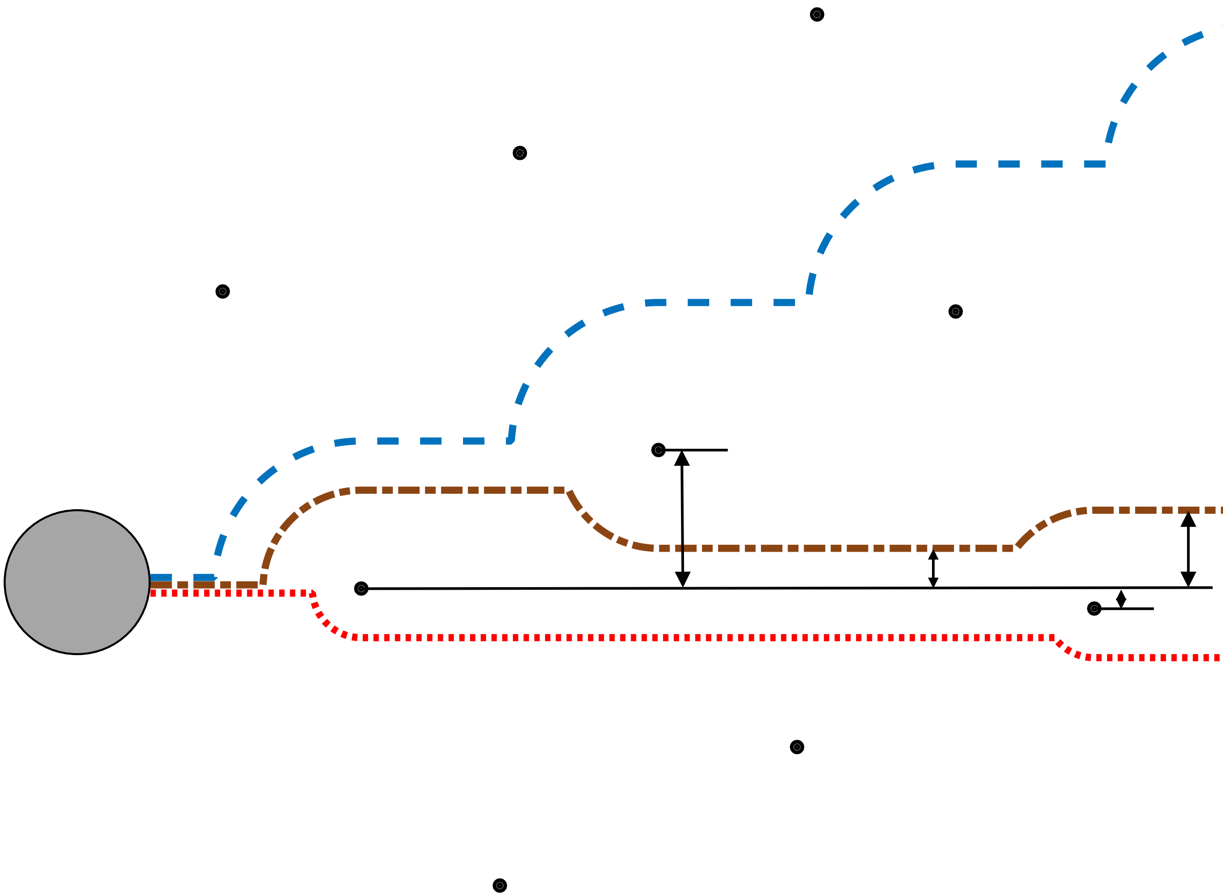
# After determining which orbits are possible, we characterized trajectories that were previously not fully understood





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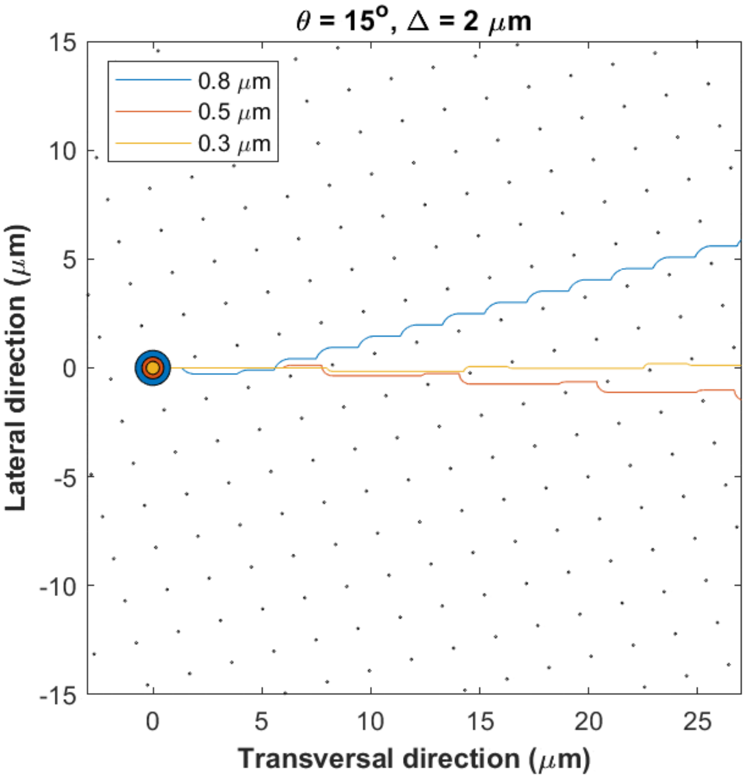
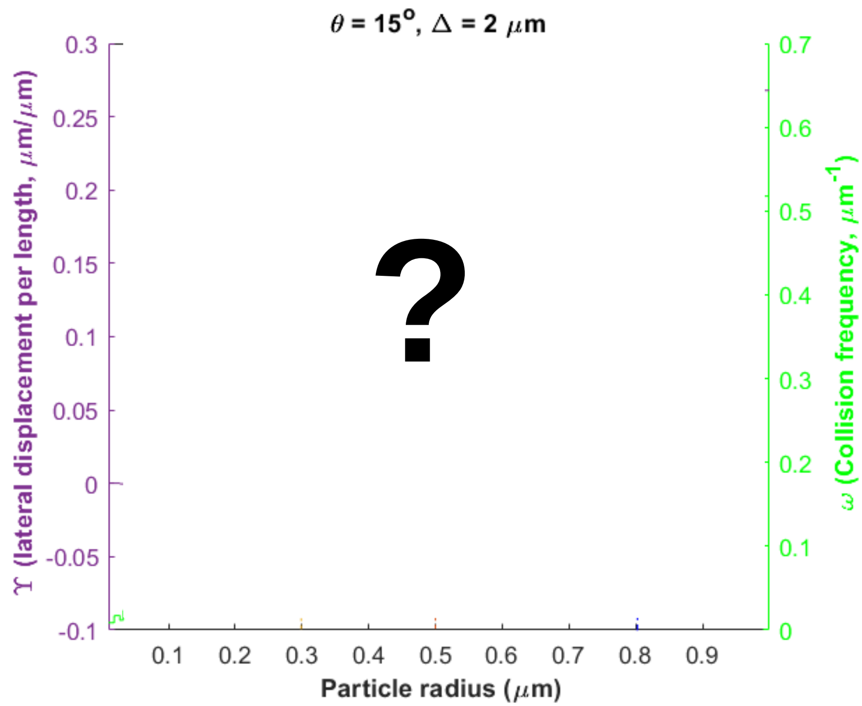
# We’ve used this restriction to fully describe all possible particle trajectories through the microdevice

- **Thanks to the symmetry constraint, we can fully describe the trajectories a particle can take through the microfluidic lattice.**



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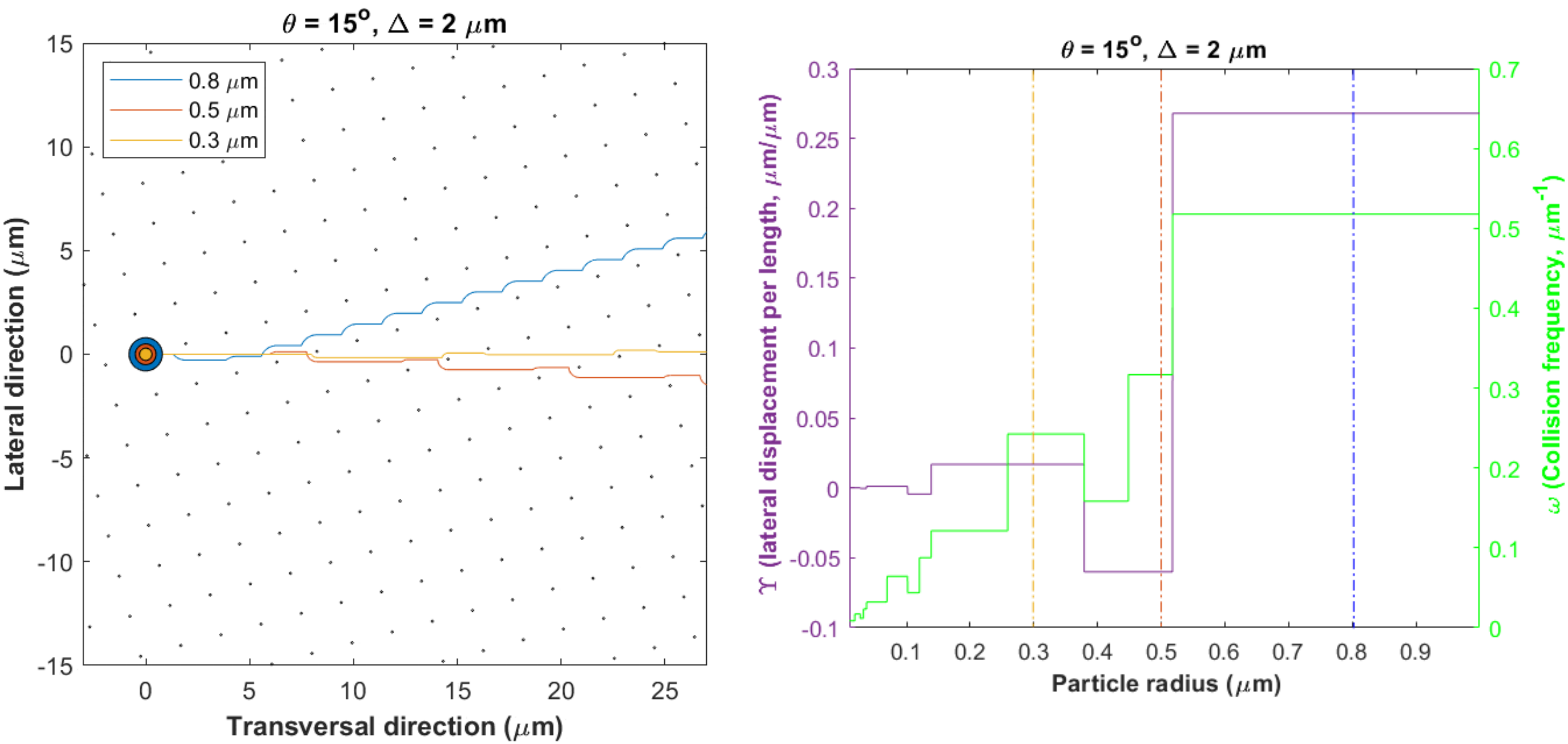




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